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NOTE

Variable Pseudoviscosity in One-Dimensional Hyperbolic Difference Schemes

One often wishes to compute the strength of the shock-wave and the fluid-flow parameters resulting from a sudden release of energy as from a high-explosive detonation or an impulsive electrical discharge. When it is desired to follow such disturbances to large distances from the source, a numerical integration of the appropriate difference equations entails use either of a very large number of mesh points or of a variable mesh spacing. Since computer memory requirements and computation time increase at least as the number of mesh points carried, it can be seen that there are compelling reasons for using a variable mesh spacing. When the von Neumann–Richtmyer pseudo-viscosity method [1] is used for following the propagation of shock waves, however, changes in mesh spacing at shock fronts give rise to spurious fluctuations in density and entropy, which do not dissipate with time. [2] We have found that variation of the pseudo-viscosity with the number of mesh points between the origin and the primary shock front can be used to eliminate these fluctuations.

The von Neumann–Richtmyer pseudoviscous term q is usually written in the form

$$q = \begin{cases} \rho(\alpha \Delta x \partial_x u)^2, & \partial_x u < 0\,, \\ 0, & \partial_x u \geqslant 0 \end{cases}$$

or

$$q = \begin{cases} \rho(\alpha \Delta u)^2, & \Delta u/\Delta x < 0\,, \\ 0, & \Delta u/\Delta x \geqslant 0 \end{cases}$$

in the difference equations, where ρ is the fluid density, u the particle velocity, and Δx the mesh spacing. The parameter α determines the number of mesh points over which a physically discontinuous shock is spread by the difference scheme. The resulting shock-front "thickness" is about $3\alpha\Delta x$.

The pseudoviscous terms have been used here in a centered-difference formulation of the Lagrangean fluid-flow equations very similar to that of Richtmyer and Morton. [3] These equations have been integrated for cylindrical and spherical shock waves in air from the strong shock region to the near-acoustic region, necessitating several increases in mesh spacing. Both ideal gas and nonideal gas

equations of state have been used. All computations were performed with a Control Data 6600 computer (which carries a 48-bit mantissa in all our calculations.)

At each time step the difference equations were integrated to about 5 mesh points beyond the primary shock front. When the pressure at the last mesh point had increased by a predetermined amount above the initial pressure, an additional point was added to the mesh. Let us define:

m = allowed maximum number of mesh points;
n = number of mesh points over which the integration is carried out at any time step;
α = pseudoviscosity parameter;
α_0 = initial value of α (in the interval $[\frac{3}{2}, 2]$);
P_0 = initial pressure of the unshocked gas;
P_n = pressure at the nth mesh point;
ΔP = allowed pressure increase at the nth mesh point.

When $P_n > P_0 + \Delta P$, the value of n is increased by one for the next time step, and α is determined by

$$\alpha = \begin{cases} \alpha_0(2n/m), & m/2 < n \leqslant m, \\ \alpha_0, & n \leqslant m/2. \end{cases}$$

When $P_m \geqslant P_0 + \Delta P$, the mesh spacing Δx is doubled for the next time step, n set equal to $m/2$, and α reset to α_0. The thermodynamic and fluid-flow parameters for the new mesh are computed from the values for the old so as to conserve mass and energy.

With the customary usage of the pseudoviscosity technique, with $\alpha = \alpha_0$ at all time, shock waves are "smeared" over a constant number of mesh points. When the mesh spacing is then doubled, the shock is momentarily compressed into half the number of mesh points, and the difference scheme reacts with spurious fluctuations in some variables which do not dissipate. With the variable pseudoviscous term, the ratio of the shock "thickness" at one time step to the "thickness" at the preceding time step is at most $(1 + 2/m)$. This ratio is unity when the mesh size is doubled, since α is halved when Δx is doubled. The shock thus occupies a constant fraction of the mesh in use throughout the calculation. With $m = 500$ in our calculation, no fluctuations in pressure, density, or temperature were observed. Furthermore, there were no cumulative errors which we could detect arising from the gradual increase in the pseudoviscosity term. A computation of the spherical shock wave from a point source of energy gave essentially exact agreement with the work of Brode [4] who used a somewhat more complicated scheme for enlarging the dimensions of the mesh.

References

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